

### Trigonomeetria põhivalemid:

$$\sin^2 \alpha + \cos^2 \alpha = 1, \quad \tan \alpha = \frac{\sin \alpha}{\cos \alpha}, \quad \cot \alpha = \frac{\cos \alpha}{\sin \alpha}$$

$$\tan \alpha \cdot \cot \alpha = 1, \quad 1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha}, \quad 1 + \cot^2 \alpha = \frac{1}{\sin^2 \alpha}$$

Mõnede nurkade trigonomeetriliste funktsioonide väärtused				
	$\sin \alpha$	$\cos \alpha$	$\tan \alpha$	$\cot \alpha$
$0^\circ$	0	1	0	puudub
$30^\circ$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$60^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$90^\circ$	1	0	puudub	0

### Täiendusnurga valemid:

$$\sin(90^\circ - \alpha) = \cos \alpha, \quad \cos(90^\circ - \alpha) = \sin \alpha$$

$$\tan(90^\circ - \alpha) = \cot \alpha, \quad \cot(90^\circ - \alpha) = \tan \alpha$$

### Negatiivse nurga trigonomeetrilised funktsioonid:

$$\sin(-\alpha) = -\sin \alpha, \quad \cos(-\alpha) = \cos \alpha$$

$$\tan(-\alpha) = -\tan \alpha, \quad \cot(-\alpha) = -\cot \alpha$$

### Põhilised taandamisvalemid

$$\sin(360^\circ \cdot n + \alpha) = \sin \alpha$$

$$\cos(360^\circ \cdot n + \alpha) = \cos \alpha$$

$$\tan(360^\circ \cdot n + \alpha) = \tan \alpha$$

$$\sin(180^\circ - \alpha) = \sin \alpha, \quad \sin(180^\circ + \alpha) = -\sin \alpha, \quad \sin(360^\circ - \alpha) = -\sin \alpha$$

$$\cos(180^\circ - \alpha) = -\cos \alpha, \quad \cos(180^\circ + \alpha) = -\cos \alpha, \quad \cos(360^\circ - \alpha) = \cos \alpha$$

$$\tan(180^\circ - \alpha) = -\tan \alpha, \quad \tan(180^\circ + \alpha) = \tan \alpha, \quad \tan(360^\circ - \alpha) = -\tan \alpha$$

$$\begin{array}{|c|c|} \hline + & + \\ \hline - & - \\ \hline \end{array}$$

$\sin \alpha$

$$\begin{array}{|c|c|} \hline - & + \\ \hline - & + \\ \hline \end{array}$$

$\cos \alpha$

$$\begin{array}{|c|c|} \hline - & + \\ \hline + & - \\ \hline \end{array}$$

$\tan \alpha$  ja  $\cot \alpha$

### Kahekordse nurga valemid:

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha, \quad \cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha, \quad \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

### Summa ja vahe siinus ning koosinus:

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta, \quad \cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

### Poolnurga trigonomeetrilised funktsioonid

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}, \quad \cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}, \quad \tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}}$$

$$\tan \frac{\alpha}{2} = \frac{\sin \alpha}{1 + \cos \alpha} = \frac{1 - \cos \alpha}{\sin \alpha}$$

### Trigonomeetriliste funktsioonide summa ja vahe teisendamine korrutiseks:

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}, \quad \sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}, \quad \cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

$$\tan \alpha \pm \tan \beta = \frac{\sin(\alpha \pm \beta)}{\cos \alpha \cos \beta}$$

### Trigonomeetriliste funktsioonide korrutise teisendamine summaks:

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)], \quad \sin^2 \alpha = \frac{1}{2} (1 - \cos 2\alpha)$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)], \quad \cos^2 \alpha = \frac{1}{2} (1 + \cos 2\alpha)$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha - \beta) - \sin(\alpha + \beta)]$$

### Trigonomeetriliste põhivõrrandite lahendamine:

Kui  $\sin x = m$ , siis  $x = (-1)^n \arcsin m + n\pi$ , kus  $n \in \mathbb{Z}$ .

Kui  $\cos x = m$ , siis  $x = \pm \arccos m + 2n\pi$ , kus  $n \in \mathbb{Z}$ .

Kui  $\tan x = m$ , siis  $x = \arctan m + n\pi$ , kus  $n \in \mathbb{Z}$ .

Kui  $\cot x = m$ , siis  $x = \operatorname{arccot} m + n\pi$ , kus  $n \in \mathbb{Z}$ .

### Võrrandid ja võrrandisüsteemid

Lineaarvõrrandi  $ax + b = 0$  lahend on  $x = -\frac{b}{a}$ .

Taandatud ruutvõrrandi  $x^2 + px + q = 0$  lahendivalem:  $x_{1,2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$ .

Taandamata ruutvõrrandi  $ax^2 + bx + c = 0$ , kus  $a \neq 1$  lahendivalem:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**Viete'i valemid:**  $x_1 + x_2 = -p$  ja  $x_1 \cdot x_2 = q$ , kus  $x_1$  ja  $x_2$  on taandatud ruutvõrrandi lahendid.

Kui  $x_1$  ja  $x_2$  on ruutvõrrandi  $ax^2 + bx + c = 0$  lahendid, siis vastav ruutkolmliige lahutub teguriteks nii:  $ax^2 + bx + c = a(x - x_1)(x - x_2)$ .

### Logaritmid omadused

$$\log_a c = b \Leftrightarrow a^b = c.$$

$$a^{\log_a c} = c.$$

$$\log_a a^x = x.$$

$$\log_a 1 = 0, \text{ kui } a > 0 \text{ ja } a \neq 1.$$

$$\log_a a = 1, \text{ kui } a > 0 \text{ ja } a = 1.$$

$$\log_a (b \cdot c) = \log_a b + \log_a c.$$

$$\log_a b^n = n \log_a b.$$

$$\log_a \frac{b}{c} = \log_a b - \log_a c.$$

$$\log_a \sqrt[n]{b} = \frac{1}{n} \log_a b.$$

$$\log_a x = \frac{\log_b x}{\log_b a}.$$

$$\log_a b = \frac{1}{\log_b a}.$$

**Kaherealine determinant:**  $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$

**Kolmerealine determinant:**  $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + bfg + cdh - gec - ahf - bdi.$

$c' = 0$	$x' = 1$	$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$
$(\sqrt{x})' = \frac{1}{2\sqrt{x}}$	$(x^n)' = nx^{n-1}$	$(e^x)' = e^x$
$(a^x)' = a^x \ln a$	$(\ln x)' = \frac{1}{x}$	$(\log_a x)' = \frac{1}{x \ln a}$
$(\sin x)' = \cos x$	$(\cos x)' = -\sin x$	$(\tan x)' = \frac{1}{\cos^2 x}$
$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$	$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$	$(\arctan x)' = \frac{1}{1+x^2}$
$\int 0 dx = C$	$\int dx = x + C$	$\int \frac{dx}{x^2} = -\frac{1}{x} + C$
$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + C$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$	$\int e^x dx = e^x + C$
$\int a^x dx = \frac{a^x}{\ln a} + C$	$\int \frac{1}{x} dx = \ln x  + C$	$\int \frac{dx}{\sin^2 x} = -\cot x + C$
$\int \cos x dx = \sin x + C$	$\int \sin x dx = -\cos x + C$	$\int \frac{dx}{\cos^2 x} = \tan x + C$
$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$	$\int \frac{dx}{\sqrt{1-x^2}} = -\arccos x + C$	$\int \frac{dx}{1+x^2} = \arctan x + C$

Kui  $\int f(x) dx = F(x) + C$ , siis  $\int f(ax+b) dx = \frac{1}{a} F(ax+b) + C$ .

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C. \quad \int [f(x)]^n \cdot f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C, \text{ kui } n \neq -1.$$

**Integreerimine muutujavahetusega:**  $\int f(x) dx = \int f[\varphi(t)] \cdot \varphi'(t) dt$ , kui  $x = \varphi(t)$ .

**Ositi integreerimine:**  $\int f(x)g'(x) dx = f(x)g(x) - \int g(x)f'(x) dx$ .

**Newton-Leibnizi valem:**  $\int_a^b f(x) dx = F(b) - F(a)$ ,  $F'(x) = f(x)$

$$\int_a^b f(x) dx = -\int_b^a f(x) dx. \quad \int_a^c f(x) dx = \int_a^b f(x) dx + \int_b^c f(x) dx.$$

**Kui funktsioon  $f(x)$  on paarisfunktsioon, siis**  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ .

**Kui funktsioon  $f(x)$  on paaritu funktsioon, siis**  $\int_{-a}^a f(x) dx = 0$ .

**Kõvertrapetsi pöörlemisel ümber  $x$ -telje tekkiva pöördkeha ruumala:**  $V = \pi \int_a^b f^2(x) dx$ .

## Vektorid

	Tasandil	Ruumis
Punktide A ja B koordinaadid	$A(x_1; y_1)$ ja $B(x_2; y_2)$	$A(x_1; y_1; z_1)$ ja $B(x_2; y_2; z_2)$
Vektori $\vec{AB}$ koordinaadid	$\vec{AB} = (x_2 - x_1; y_2 - y_1)$	$\vec{AB} = (x_2 - x_1; y_2 - y_1; z_2 - z_1)$
Vektori pikkus	$ \vec{AB}  = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$ \vec{AB}  = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
Vektorite $\vec{u}$ ja $\vec{v}$ koordinaadid	$\vec{u} = (u_x; u_y)$ $\vec{v} = (v_x; v_y)$	$\vec{u} = (u_x; u_y; u_z)$ $\vec{v} = (v_x; v_y; v_z)$
Vektorite summa ja vahe	$\vec{u} \pm \vec{v} = (u_x \pm v_x; u_y \pm v_y)$	$\vec{u} \pm \vec{v} = (u_x \pm v_x; u_y \pm v_y; u_z \pm v_z)$
Vektori korrutis arvuga $r$	$r \cdot \vec{u} = (ru_x; ru_y)$	$r \cdot \vec{u} = (ru_x; ru_y; ru_z)$
Vektorite $\vec{u}$ ja $\vec{v}$ skalaarkorrutis	$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y$	$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z$
Vektorite $\vec{u}$ ja $\vec{v}$ vaheline nurk	$\cos \varphi = \frac{\vec{u} \cdot \vec{v}}{ \vec{u}   \vec{v} }$	
Vektorite $\vec{u}$ ja $\vec{v}$ kollineaarsus	$\frac{u_x}{v_x} = \frac{u_y}{v_y}$	$\frac{u_x}{v_x} = \frac{u_y}{v_y} = \frac{u_z}{v_z}$

Ruumivektorite  $\vec{u}$  ja  $\vec{v}$  **vektorkorrutis:**  $\vec{u} \times \vec{v} = \begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} = \begin{vmatrix} u_x & u_z \\ v_x & v_z \end{vmatrix} \begin{vmatrix} u_y & u_z \\ v_y & v_z \end{vmatrix} \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$

Kolme vektori  $\vec{a} = (a_x; a_y; a_z)$ ,  $\vec{b} = (b_x; b_y; b_z)$  ja  $\vec{c} = (c_x; c_y; c_z)$  **segakorrutis:**

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

**Vektorite  $\vec{a}$ ,  $\vec{b}$  ja  $\vec{c}$  komplanaarsuse tunnus:**  $\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = 0$ .

## Tasandi võrrandid

Üldvõrrand  $Ax + By + Cz + D = 0$ , kus  $\vec{n} = (A; B; C)$ .

Punkti  $P_1(x_1; y_1; z_1)$  ja normaalvektoriga määratud tasandi võrrand  $(x - x_1) \cdot A + (y - y_1) \cdot B + (z - z_1) \cdot C = 0$ .

Tasandi võrrand, kui tasand on määratud

punkti  $P_1(x_1; y_1; z_1)$  ja kahe mittekollineaarse punktidega  $P_1(x_1; y_1; z_1)$ ,  $P_2(x_2; y_2; z_2)$  ja vektoriga  $\vec{a} = (a_x; a_y; a_z)$  ja  $\vec{b} = (b_x; b_y; b_z)$  punktidega  $P_1(x_1; y_1; z_1)$ ,  $P_2(x_2; y_2; z_2)$  ja  $P_3(x_3; y_3; z_3)$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = 0. \quad \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0.$$

Punkti  $P_1(x_1; y_1; z_1)$  kaugus tasandist  $Ax + By + Cz + D = 0$   $d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$ .