

Ulesandeis 14.20–14.35 leida antud lineaarse homogeense vörrandi üldlahend, kui on teada tema mks erilahend y_1 . Nenies ülesannetesi, kus erilahendit pole antud, võib erilahendi leida vörrandi kujust lähtudes, otsides teda eksponentfunktsiooni $y_1 = e^{ax}$ vki poldnoomi $y_1 = x^n + ax^{n-1} + bx^{n-2} + \dots$ kujul.

$$14.20. (2x+1)y'' + 4xy' - 4y = 0.$$

$$14.21. x(x-1)y'' - xy' + y = 0.$$

$$14.22. x^2y''\ln x - xy' + y = 0.$$

$$14.23. xy'' - (2x+1)y' + 2y = 0.$$

$$14.24. (x^2+1)y'' - 2y = 0.$$

$$14.25. (x^2+1)y'' + xy' - y = 0.$$

$$14.26. xy'' - (x+1)y' - 2(x-1)y = 0.$$

$$14.27. y'' + \frac{2}{x}y' + y = 0, y_1 = \frac{\sin x}{x}.$$

$$14.28. xy'' + 2y' - xy = 0, y_1 = e^x/x.$$

$$14.29. y'' - 2(1+\tan^2 x)y = 0, y_1 = \tan x.$$

$$14.30. (e^x+1)y'' - 2y' - e^x y = 0, y_1 = e^x - 1.$$

$$14.31. y'' + y'(\tan x) - y \cos^2 x = 0, y_1 = e^{\sin x}.$$

$$14.32. y'' + (\tan x - 2\tan x)y' + 2y(\cot x)^2 = 0, y_1 = \sin x.$$

$$14.33. y'' - \frac{2y}{\cos x} = 0, y_1 = \tan x.$$

$$14.34. x^2y'' - 2xy' + (4x^2+2)y = 0, y_1 = x \cos 2x.$$

$$14.35. y'' + y'(\tan x) + y \cos^2 x = 0, y_1 = \cos(\sin x).$$

Ülesandeis 14.36–14.39 leida antud lineaarse mittehomogeense vörrandi üldlahend, kui on teada temale vastava homogeense vörrandi mks erilahend y_1 .

$$14.36. y'' + y' + e^{-2x} \cdot y = e^{-3x}, y_1 = \cos e^{-x}.$$

$$14.37. y'' - y' + e^{2x} \cdot y = xe^{2x} - 1, y_1 = \sin e^x.$$

$$14.38. y'' + \frac{2}{x}y' + y = \frac{1}{x}, y_1 = \frac{\sin x}{x}.$$

$$14.39. x^2y'' - xy' - 3y = 5x^4, y_1 = 1/x.$$

Lahendada lineaarsed mittehomogeensed diferentsiaalvörrandid 14.40–14.45.

$$14.40. xy'' - (1+2x^2)y' = 4x^2e^x.$$

$$14.41. y'' - 2y'tan x = 1.$$

$$14.42. y''x(\ln x) - y' = \ln^2 x.$$

$$14.43. xy'' + (2x-1)y' = -4x^2.$$

$$14.44. y'' + y'\tan x = (\cos x) \cdot (\cot x).$$

$$14.45. (2x+1)y'' + (2x-1)y' - 2y = x^2 + x.$$

Ülesandeis 1241–1255 leida diferentsiaalvörrandi üldlahend.

$$1241. y'' = x^2 + \ln x.$$

$$1242. \frac{d^2y}{dx^2} = 1+x^2.$$

$$1243. y'' = x + \arctan x.$$

$$1244. y'' = x + \sin x.$$

$$1245. y'' = \ln x.$$

$$1246. x^2y'' = y'^2.$$

$$1247. (1+x)\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0.$$

$$1248. xy'' + y' - 4x = 0.$$

$$1249. x^2y'' + xy' = 1.$$

$$1250. (1+x^2)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 1 = 0.$$

$$1251. y\frac{d^2y}{dx^2} = y^2 \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2.$$

$$1252. y'' \tan y - 2y'^2 = 0.$$

$$1253. yy'' + y'^2 = y'.$$

$$1254. y\frac{d^2y}{dx^2} = 1 + \left(\frac{dy}{dx}\right)^2.$$

$$1255. (1-y)y'' + 2y'^2 = 0.$$

Ülesandeis 1256–1270 leida teist järku lineaarse konstantsete kordajatega homogeense diferentsiaalvörrandi üldlahend.

$$1256. y'' - 7y' + 12y = 0.$$

$$1260. y'' = 4y' + y.$$

$$1257. 2y'' + 3y' = 2y.$$

$$1258. y'' - 7y' = 0.$$

$$1261. y'' + \frac{1}{2}y' + \frac{1}{16}y = 0.$$

$$1259. 3y'' - 4y' = 0.$$

$$1262. 9y'' - 6y' + y = 0.$$

$$1267. y'' + 7y' + 17y = 0.$$

$$1263. 36y'' - 12y' + y = 0.$$

$$1268. 2y'' + 5y = 0.$$

$$1264. y'' - 6y' + 9y = 0.$$

$$1269. 2y'' + 4y' + 5y = 0.$$

$$1265. 4y'' + 4y' + y = 0.$$

$$1270. y'' = 8y' - 25y.$$

$$1266. y'' + 9y = 0.$$

Ülesandeis 1271–1275 leida diferentsiaalvörrandi erilahend, mis rahuldab algtingimusi.

$$1271. 3y'' = 5y', y(0) = 2, y'(0) = -\frac{5}{3}.$$

$$1272. 3y'' - y' = 0, y(3) = 1, y'(3) = \frac{1}{3}.$$

$$1273. y'' - 6y' + 9y = 0, y(1) = 0, y'(1) = e^3.$$

$$1274. y'' = 2(3y' - 5y), y(0) = 1, y'(0) = 4.$$

$$1275. y'' = 5y - 4y', y(-1) = 0, y'(-1) = 1.$$

Integreerida järgmised lihtsamad teist järku diferentsiaalvörrandid, mis taanduvad esimest järku vörrandeiks.

$$118. xy''' = 2. Vast. y = x^2 \ln x + C_1x^2 + C_2x + C_3; \text{ leida algtingimus } x = 1, y = 1, y' = 1, y'' = 3 \text{ rahulday erilahend.}$$

$$119. y(n) = x^m. Vast. y = \frac{mx^{m+n}}{(m+n)!} + C_1x^{n-1} + \dots + C_{n-1}x + C_n.$$

$$120. y'' = a^2y. Vast. ax = \ln(ay + \sqrt{a^2y^2 + C_1}) + C_2 \text{ ehk } y = C_1ea^x + C_2e^{-ax}.$$

$$121. y'' = \frac{a}{y^3}. Vast. (C_1x + C_2)^2 = C_1y^2 - a.$$

Näidetes 122–125 leida algtingimustele $x = 0, y = -1, y' = 0$ vastav erilahend. 122. $xy'' - y' = x^2e^x. Vast. y = ex(x-1) + C_1x^2 + C_2. Eri lahend. y = ex(x-1). 123. yu'' - y'^2 + y^3 = 0. Vast. y + C_1 \ln y = x + C_2. Eri lahend. y = -1. 124. y'' + y' \tan x = \sin 2x. Vast. y = C_2 + C_1 \sin x - x - \frac{1}{2} \sin 2x. Eri lahend. y = 2 \sin x - \sin x \cos x - x - 1. 125. (y'')^2 + (y')^2 = a^2. Vast. y = C_2 - a \cos(x + C_1). Eri lahendid: y = a - 1 - a \cos x; y = a \cos x - (a+1). (Näpunaide. Parameetriline kuju on y'' = a \cos t, y' = a \sin t.)$

$$126. y'' = \frac{1}{2y^2}. Vast. y = \pm \frac{2}{3}(x + C_1)^{\frac{3}{2}} + C_2.$$

$$127. y''' = y'^2. Vast. y = (C_1 - x)[\ln(C_1 - x) - 1] + C_2x + C_3.$$

$$128. y''y''' - 3y'^2 = 0. Vast. x = C_1y^2 + C_2y + C_3.$$

Integreerida järgmised konstantsete kordajatega lineaarsed diferentsiaalvörrandid.

$$129. y'' = 9y. Vast. y = C_1e^{3x} + C_2e^{-3x}.$$

$$130. y'' + y = 0. Vast. y = A \cos x + B \sin x.$$

$$131. y'' - y' = 0. Vast. y = C_1 + C_2ex.$$

$$132. y'' + 12y = 7y'. Vast. y = C_1e^{3x} + C_2e^{4x}.$$

$$133. y'' - 4y' + 4y = 0. Vast. y = (C_1 + C_2x)e^{2x}.$$

$$134. y'' + 2y' + 10y = 0. Vast. y = e^{3x}(A \cos 3x + B \sin 3x).$$

$$135. y'' + 3y' - 2y = 0. Vast. y = C_1e^{-\frac{3}{2}x} + C_2e^{-\frac{1}{2}x}.$$

$$136. 4y'' - 12y' + 9y = 0. Vast. y = (C_1 + C_2x)e^{\frac{3}{2}x}.$$

$$137. y'' + y' + y = 0. Vast. y = e^{-\frac{1}{2}x} \times [A \cos(\sqrt{\frac{3}{2}}x) + B \sin(\sqrt{\frac{3}{2}}x)].$$

$$140. y^{IV} - 5y'' + 4y = 0. Vast. y = C_1e^x + C_2e^{-x} + C_3e^{2x} + C_4e^{-2x}.$$

$$141. y'' - 2y' - y' + 2y = 0. Vast. y = C_1e^{2x} + C_2e^x + C_3e^{-x}.$$

$$142. y'' - 3ay'' + 3a^2y' - a^2y = 0. Vast. y = (C_1 + C_2x + C_3x^2)e^{ax}.$$

$$143. yV - 4y'' = 0. Vast. y = C_1 + C_2x + C_3x^2 + C_4e^{2x} + C_5e^{-2x}.$$

$$144. y^{IV} - 2y'' + 9y = 0. Vast. y = (C_1 \cos \sqrt{2x} + C_2 \sin \sqrt{2x})e^{-x} + (C_3 \cos \sqrt{2x} + C_4 \sin \sqrt{2x})ex.$$

$$145. y^{IV} - 8y'' + 16y = 0. Vast. y = C_1e^{2x} + C_2e^{-2x} + C_3xe^{2x} + C_4xe^{-2x}.$$

$$146. y^{IV} + y = 0. Vast. y = e^{\frac{x}{\sqrt{2}}} \left(C_1 \cos \frac{x}{\sqrt{2}} + C_2 \sin \frac{x}{\sqrt{2}} \right) + e^{-\frac{x}{\sqrt{2}}} \left(C_3 \cos \frac{x}{\sqrt{2}} + C_4 \sin \frac{x}{\sqrt{2}} \right).$$

$$147. y^{IV} - a^2y = 0. Leida üldlahend ja see erilahend, mis rahuldab algtingimusi $y = 1, y' = 0, y'' = 0, y''' = 0$, kui $x_0 = 0$. Vast. Üldlahend $y = C_1e^{ax} + C_2e^{-ax} + C_3 \cos ax + C_4 \sin ax$. Eri lahend: $u_0 = \cos ax$.$$

Ülesandeis 1276–1295 leida teist järku lineaarse konstantsete kordajatega mittehomogeense diferentsiaalvörrandi üldlahend.

$$1276. y'' - 7y' + 12y = x.$$

$$1277. y'' + y' = 8x^3 + 24x^2 - 10x.$$

$$1278. y'' - 3y' + 9x^2 + 6x - 13 = 0.$$

$$1279. y'' - 4y' - 5y = x^3 + 1.$$

$$1280. \frac{d^2y}{dx^2} + 4y = 2x + 3.$$

$$1281. \frac{d^2y}{dx^2} - 2y = 4e^{2x}.$$

$$1282. \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = 3e^{-x}.$$

$$1283. y'' - 2y' - 3y = e^{-x}.$$

$$1284. y'' - 6y' + 13y = e^{-2x}.$$

$$1285. y'' + 3y' + 2y = 6e^{-5x}.$$

$$1286. 7 \frac{d^2y}{dx^2} + 6 \frac{dy}{dx} + 5y = x \sin x.$$

$$1287. y'' - 7y' + 6y = \sin x.$$

$$1288. \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} - y = \sin x.$$

$$1289. y'' + 2y' + 5y = 2x \sin 2x.$$

$$1290. \frac{d^2y}{dx^2} + 9y = \sin 3x.$$

$$1291. \frac{d^2y}{dx^2} + y = 2x^3 - x + 2 + \cos x.$$

$$1292. y'' - 2y' + y = 1 + x + 2(3x^2 - 2)e^x.$$

$$1293. \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 3y = x + e^{2x}.$$

$$1294. \frac{d^2y}{dx^2} + 3y = \sin x + \frac{1}{3} \sin 3x.$$

$$1295. \frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = x^2 e^{4x} + 2e^{4x}.$$

Ülesandeis 1296–1300 leida diferentsiaalvörrandi üldlahend konstantide varieerimise meetodi abil.

$$1296. y'' + 4y' + 4y = \frac{e^{-2x}}{x^3}.$$

$$1297. y'' + y = \frac{1}{\cos x}.$$

$$1298. y'' - y' = \frac{ex}{3 - ex}.$$

$$1299. y'' + y = \tan x.$$

$$1300. y'' - 2y' + y = \frac{ex}{x^2 + 1}.$$

Ülesandeis 1301–1305 leida kõrgemat järku lineaarse konstantsete kordajatega diferentsiaalvörrandi üldlahend.

$$1301. y'' - 7y' + 6y = \sin x.$$

$$1302. y'' + 6y' + 11y' + 6y = 6x^3 + 2x^2 + 1.$$

$$1303. y'' - y' = x + 1.$$

$$1304. y^{IV} - 4y = e^x.$$

$$1305. y'' + y'' = x^2.$$

Integreerida järgmised lineaarsed mittehomogeensed vörandid vörrandide üldlahend.

$$148. y'' - 7y' + 12y = x. Vast. y = C_1e^{3x} + C_2e^{4x} + \frac{7x + 7}{144}.$$

$$149. s'' - a^2s = t + 1. Vast. s = C_1e^{at} + C_2e^{-at} - \frac{t+1}{a^2}.$$

$$150. y'' + y' - 2y = 8 \sin 2x. Vast. y = C_1e^{x} + C_2e^{-x} + \frac{1}{2}(\sin 2x + 2 \cos 2x).$$

$$14.21. y = C_1x + C_2x^2 + C_3x^3 + C_4x^4 + C_5x^5 + \dots$$

$$14.22. y = C_1 \cos(\sin x) + C_2 \sin(\sin x) + C_3 \cos(\cos x) + C_4 \sin(\cos x) + C_5 \cos(\sin x + \pi) + C_6 \sin(\cos x + \pi) + \dots$$

$$14.23. y = C_1 \cos(e^x) + C_2 \sin(e^x) + C_3 \cos(e^{-x}) + C_4 \sin(e^{-x}) + C_5 \cos(e^{2x}) + C_6 \sin(e^{2x}) + C_7 \cos(e^{-2x}) + C_8 \sin(e^{-2x}) + \dots$$

$$14.24. y = C_1 \sin x + C_2 \cos x + C_3 \sin x + C_4 \cos x + C_5 \sin x + C_6 \cos x + \dots$$

$$14.25. y = C_1 \cos(\sin x) + C_2 \sin(\sin x) + C_3 \cos(\cos x) + C_4 \sin(\cos x) + C_5 \cos(\sin x + \pi) + C_6 \sin(\cos x + \pi) + \dots$$

$$14.26. y = C_1 \cos(\sin x) + C_2 \sin(\sin x) + C_3 \cos(\cos x) + C_4 \sin(\cos x) + C_5 \cos(\sin x + \pi) + C_6 \sin(\cos x + \pi) + \dots$$

$$14.27. y = C_1 \cos(e^x) + C_2 \sin(e^x) + C_3 \cos(e^{-x}) + C_4 \sin(e^{-x}) + C_5 \cos(e^{2x}) + C_6 \sin(e^{2x}) + C_7 \cos(e^{-2x}) + C_8 \sin(e^{-2x}) + \dots$$

$$14.28. y = C_1 \sin x + C_2 \cos x + C_3 \sin x + C_4 \cos x + C_5 \sin x + C_6 \cos x + \dots$$

$$14.29. y = C_1 \cos(\sin x) + C_2 \sin(\sin x) + C_3 \cos(\cos x) + C_4 \sin(\cos x) + C_5 \cos(\sin x + \pi) + C_6 \sin(\cos x + \pi) + \dots$$

$$14.30. y = C_1 \sin x + C_2 \cos x + C_3 \sin x + C_4 \cos x + C_5 \sin x + C_6 \cos x + \dots$$

$$14.31. y = C_1 \cos(e^x) + C_2 \sin(e^x) + C_3 \cos(e^{-x}) + C_4 \sin(e^{-x}) + C_5 \cos(e^{2x}) + C_6 \sin(e^{2x}) + C_7 \$$

$$\begin{aligned}
& + \frac{C}{1+C^2}, \text{ singulaarne laehend } x = \frac{p^2-1}{(p^2+1)^2}, y = \frac{p^3}{(p^2+1)^2}. & 1231. \quad y = 3x^2. \\
1232. \quad x+y^2+Cy=0. & 1233. \quad x=Cy \pm \frac{a^2}{y}. & 1234. \quad y=Cx+\frac{a^2}{2x}. & 1235. \quad y= \\
& = Cxe^{-x}. & 1236. \quad y^2=\ln Cx^{2k}, \text{ kus } k \text{ on vordetegur.} & 1237. \quad \ln|Cx|+2\sqrt{\frac{y}{x}}=0. \\
1238. \quad x^2=C^2+2Cy. & 1239. \quad y=\frac{x}{k-Cx}, \text{ kus } k \text{ on vordetegur.} & 1240. \quad y= \\
& = -x\ln|x|+Cx. & 1241. \quad y=\frac{x^2}{2}\ln x+\frac{x^4}{12}-\frac{x}{4}+C_1x+C_2. & 1242. \quad y=x\ln x+ \\
& + \frac{1}{6}x^3+C_1x+C_2. & 1243. \quad y=\frac{\arctan x}{2}(x^2-1)-\frac{x}{2}\ln(1+x^2)+\frac{x^3}{6}+C_1x+C_2. \\
1244. \quad y=\frac{x^3}{6}-\sin x+C_1x+C_2. & 1245. \quad y=\frac{x^2}{2}\left(\ln x-\frac{3}{2}\right)+C_1x+C_2. \\
1246. \quad y=C_1x-C_1^2\ln|x|+C_1+C_2. & 1247. \quad y=C_1\ln|1+x|+C_2. & 1248. \quad y=x^2+ \\
& + C_1\ln|x|+C_2. & 1249. \quad y=\frac{1}{2}(\ln x)^2+C_1\ln x+C_2. & 1250. \quad y=(1+C_1^2)\ln(C_1+ \\
& + x)-C_1x+C_2. & 1251. \quad y=\frac{C_1C_2e^{C_1x}}{1-C_2e^{C_1x}}. & 1252. \quad \cot y=C_2-C_1x. & 1253. \quad x= \\
& = C_1\ln(y-C_1)+y+C_2. & 1254. \quad y=\frac{C_1}{2}\left(e^{\frac{x-c_2}{c_1}}+e^{-\frac{x-c_2}{c_1}}\right). & 1255. \quad y=1- \\
& - \frac{1}{C_1x+C_2}. & 1256. \quad y=C_1e^{3x}+C_2e^{4x}. & 1257. \quad y=C_1e^{\frac{x}{2}}+C_2e^{-2x}. & 1258. \quad y=C_1+C_2e^{7x}. \\
1259. \quad y=C_1+C_2e^{\frac{4x}{3}}. & 1260. \quad y=e^{2x}\left(C_1e^{\frac{\sqrt{5}x}{2}}+C_2e^{-\frac{\sqrt{5}x}{2}}\right). & 1261. \quad y=(C_1+C_2x)e^{-\frac{x}{4}}. \\
1262. \quad y=(C_1+C_2x)e^{\frac{x}{3}}. & 1263. \quad y=(C_1+C_2x)e^{\frac{x}{6}}. & 1264. \quad y=(C_1+C_2x)e^{3x}. \\
1265. \quad y=(C_1+C_2x)e^{-\frac{x}{2}}. & 1266. \quad y=C_1\cos 3x+C_2\sin 3x. & 1267. \quad y= \\
& - e^{-\frac{7}{2}x}\left(C_1\cos \frac{\sqrt{19}}{2}x+C_2\sin \frac{\sqrt{19}}{2}x\right). & 1268. \quad y=C_1\cos \sqrt{\frac{5}{2}}x+C_2\sin \sqrt{\frac{5}{2}}x. \\
1269. \quad y=e^{-x}\left(C_1\cos \frac{\sqrt{6}}{2}x+C_2\sin \frac{\sqrt{6}}{2}x\right). & 1270. \quad y=e^{4x}(C_1\cos 3x+C_2\sin 3x). \\
1271. \quad y=e^{\frac{5}{3}x}+1. & 1272. \quad y=\frac{e^{\frac{x}{3}-1}}{6}[e^{x+1}-e^{-5(x+1)}]. & 1273. \quad y=(x-1)e^{3x}. & 1274. \quad y= \\
& = e^{3x}(\cos x+\sin x). & 1275. \quad y=\frac{1}{6}[e^{x+1}-e^{-5(x+1)}]. & 1276. \quad y=C_1e^{3x}+C_2e^{4x}+ \\
& + \frac{1}{144}(12x+7). & 1277. \quad y=2x^4-5x^2+10x+C_1+C_2e^{-x}. & 1278. \quad y=C_1+C_2e^{3x}+ \\
& + x^3+2x^2-3x. & 1279. \quad y=C_1e^{5x}+C_2e^{-x}-\frac{x^3}{5}+\frac{12x^2}{25}-\frac{126x}{125}-\frac{499}{625}. \\
1280. \quad y=C_1\cos 2x+C_2\sin 2x+\frac{1}{4}(2x+3). & 1281. \quad y=C_1e^{-x}+e^{2x}\left(C_2+\frac{4}{3}x\right). \\
& + 5C_3e^{2t}. & 1312. \quad x=C_1e^t+C_2e^{-t}+C_3e^{2t}, & z=-2C_1e^t+C_2e^{-t}+ \\
& + 5C_3e^{2t}. & 1311. \quad x=C_1C_2e^{ct}, y=C_2e^{ct}, z=C_1e^t+C_2e^{-t}+ \\
& = C_1(1+\sqrt{2})e^{\sqrt{2}t}+C_2(1-\sqrt{2})e^{-\sqrt{2}t}. & 1310. \quad x=C_1e^t+C_2e^{ct}, y=C_2e^{ct}, z=C_1e^t+C_2e^{-t}+ \\
& = -C_1e^t+C_2e^{-t}-2C_3e^{2t}, y=-C_1e^t+3C_2e^{-t}-3C_3e^{2t}, z=C_1e^t-C_2e^{-t}+ \\
& + 7C_3e^{2t}. &
\end{aligned}$$