4.4. Ülesanded. Lahendada lineaarsed võrrandid 4.1-4.24.

4.1. $(1 + x^2)y' = 2xy$.

4.2. $y' - y = x - 1$.

4.3. $xy' + y = 3x^2$.

4.4. $x^2y' + xy + 1 = 0$.

4.5. $x(\ln|x|)y' - y \ln|x| = x$.

4.6. $y' + y \cos x = e^{-\sin x}$.

4.7. $y' - y \cot x = 2x \sin x$.

4.8. $y' \cos x + y \sin x = 1$.

4.9. $y = x(y' - x \cos x)$.

4.10. $y' + 2xy = 2xe^{-x^2}$.

4.11. $(xy + e^x)dx = x\,dy$.

4.12. $x\,dy + (x^2 - y)dx = 0$.

4.13. $x^2\,dy - 2xy\,dx = 3dx$.

4.14. $(x + 1)y' - 2y = (x + 1)^4$.

4.15. $(x + a)dy = 5(x + a)^3dx - 2y\,dx$ $(a = \text{const})$.

4.16. $y' + p'(x)y = p(x) \cdot p'(x)$.

4.17. $2y\,dx + (y^2 - 2x)dy = 0$.

4.18. $(x + y^2)dy = y\,dx$.

4.19. $y\,dx + (2x - 6y^4)dy = 0$.

4.20. $(2e^y - x)dy = dx$.

4.21. $(e^{-\frac{y}{2}} - xy)dy = dx$.

4.22. $(2y(\ln y) + y - x)dy = y\,dx$.

4.23. $(\sin^2 y + x \cot y)dy = dx$.

4.24. $dx + (x - \frac{e^{-y}}{\cos^2 y})dy = 0$.

Lahendada Bernoulli võrrandid 4.25-4.39.

4.25. $y' - \frac{y}{x} = \frac{1}{2y}$.

4.26. $3y^2y' + y^3 = 1$.

4.27. $y' + 2y = y^2e^x$.

4.28. $(x + 1)(y' + y^2) = -y$.

4.29. $y' + \frac{xy}{1 - x^2} = x\sqrt{y}$ $(-1 < x < 1)$.

4.30. $xy' - 2x^2\sqrt{y} = 4y$.

4.31. $y' - \frac{3}{2x}y = \frac{3x}{2}\,y^{\frac{1}{3}}$.

4.32. $y' = (y + y^2)\cos x$.

4.33. $y' = y^4(\cos^3 x) + y \tan x$.

4.34. $xy^2\,dy = (x^2 + y^3)dx$.

4.35. $xy\,dy = (y^2 + x)dx$.

4.36. $(xy + x^2y^3)dy = dx$

4.37. $(x^3 + e^y)dy = 3x^2\,dx$.

4.38. $(2x^2\,y(\ln y) - x)dy = y\,dx$.

4.39. $dy(x^3 \sin y - x) + 2y\,dx = 0$.

Integreerida Bernoulli võrrandid:

66. $y' + xy = x^3y^3$. *Vast.* $y^2(x^2 + 1 + Ce^{x^2}) = 1$.

67. $(1 + x^2)y' - xy - axy^2 = 0$. *Vast.* $(C\sqrt{1 - x^2} - a)y = 1$.

68. $3y^2y' - ay^3 - x - 1 = 0$. *Vast.* $a^2y^3 = Ce^{ax} - a(x + 1) - 1$.

69. $y'(x^2y^3 + xy) = 1$. *Vast.* $x[(2 - y^2)e^{\frac{1}{2}y^2} + C] = e^{\frac{1}{2}y^2}$.

70. $(y \ln x - 2)y\,dx = x\,dy$. *Vast.* $y(Cx + \ln x + 1) = 1$.

71. $y - y' \cos x = y^2 \cos x(1 - \sin x)$. *Vast.* $y = \frac{\tan x + \sec x}{\sin x + C}$.

Integreerida järgmised eksaktsed diferentsiaalvõrrandid.

72. $(x^2 + y)dx + (x - 2y)dy = 0$. *Vast.* $\frac{x^3}{3} + xy - y^2 = C$.

73. $(y - 3x^2)dx - (4y - x)dy = 0$. *Vast.* $2y^2 - xy + x^3 = C$.

74. $(y^3 - x)y' = y$. *Vast.* $y^4 = 4xy + C$.

75. $\left[\frac{y^2}{(x - y)^2} - \frac{1}{x}\right]dx + \left[\frac{1}{y} - \frac{x^2}{(x - y)^2}\right]dy = 0$. *Vast.* $\ln\frac{y}{x} - \frac{xy}{x - y} = C$

76. $2(3xy^2 + 2x^3)dx + 3(2x^2y + y^2)dy = 0$. *Vast.* $x^4 + 3x^2y^2 + y^3 = C$.

77. $\frac{x\,dx + (2x + y)dy}{(x + y)^2} = 0$. *Vast.* $\ln(x + y) - \frac{x}{x + y} = C$.

78. $\left(\frac{1}{x^2} + \frac{3y^2}{x^4}\right)dx = \frac{2y\,dy}{x^3}$. *Vast.* $x^2 + y^2 = Cx^3$.

79. $\frac{x^2\,dy - y^2\,dx}{(x - y)^2} = 0$. *Vast.* $\frac{xy}{x - y} = C$.

80. $x\,dx + y\,dy = \frac{y\,dx - x\,dy}{x^2 + y^2}$. *Vast.* $x^2 + y^2 - 2\arctan\frac{x}{y} = C$.

5.1. $xdy + ydx = 0$.

5.2. $x(y^2 + 1)dx + (x^2y + 2y^3)dy = 0$.

5.3. $\frac{2x}{y^3}dx + (\frac{1}{y^2} - \frac{3x^2}{y^4})dy = 0$.

5.4. $(y^3 - x)dy = ydx$.

5.5. $(\frac{1}{x^2} + \frac{3y^2}{x^4})dx = \frac{2y\,dy}{x^3}$.

5.6. $(\frac{x}{\sqrt{x^2 - y^2}} - 1)dx - \frac{y\,dy}{\sqrt{x^2 - y^2}} = 0$.

5.7. $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1 - \frac{x}{y})dy = 0$.

5.8. $e^{-y}\,dx = (2y + xe^{-y})dy$.

4.1. $y=C(1+x^2)$. 4.2. $y=Ce^x-x$. 4.3. $y=x^2+Cx^{-1}$. 4.4. $xy=C-\ln|x|$. 4.5. $y=Cx+x\ln|\ln|x||$. 4.6. $y=(x+C)e^{-\sin x}$. 4.7. $y=(x^2+C)\sin x$. 4.8. $y=\sin x+C\cos x$. 4.9. $y=x(C+\sin x)$. 4.10. $y=(x^2+C)e^{-x^2}$. 4.11. $y=e^x(C+\ln|x|);x=0$. 4.12. $y=x(C-x)$; $x=0$. 4.13. $y=Cx^2-x^{-1}$; $x=0$. 4.14. $2y=(x+1)^4+2C(x+1)^2$ $(x\neq -1)$. 4.15. $y=C(x+a)^{-2}+(x+a)^3$ $(x\neq -a)$; $x=-a$ 4.16. $y=p(x)-1+Ce^{-p(x)}$. 4.17. $2x=2Cy-y^2$; $y=0$. 4.18. $x=Cy+y^2$; $y=0$. 4.19. $x=Cy^{-2}+y^4$; $y=0$. 4.20. $x=Ce^{-y}+e^y$.

4.21. $x=(C+y)e^{-\frac{y^2}{2}}$. 4.22. $x=Cy^{-1}+y\ln y$. 4.23. $x=(C-\cos y)\sin y$. 4.24. $x=(C+\tan y)e^{-y}$. 4.25. $y^2=Cx^2-x$ 4.26. $y^3=1+Ce^{-x}$. 4.27. $y(e^x+Ce^{+2x})=1$; $y=0$. 4.28. $y(x+1)(\ln|x+1|+C)=1$; $y=0$. 4.29. $3\sqrt{y}=3C\sqrt[4]{1-x^2}-(1-x^2)$; $y=0$ $(x\neq 0)$. 4.30. $y=x^4(C+\ln|x|)^2$; $y=0$. 4.31. $y^3=Cx+x^2$; $y=0$ $(x\neq 0)$. 4.32. $y(Ce^{-\sin x}-1)=1$; $y=0$. 4.33. $y^{-3}=(C+3x)\cos^3 x$; $y=0$. 4.34. $y^3=Cx^3-3x^2$; $x=0$. 4.35. $y^2=Cx^2-2x$; $x=0$. 4.36. $x(Ce^{-\frac{y^2}{2}}-y^2+2)=4$ $x=0$. 4.37. $x^3=(C+y)e^y$. 4.38. $xy(C-\ln^2 y)=1$. 4.39. $x^2(C-\cos y)=y$; $y=0$; $x=0$.

5.1. $xy=C$. 5.2. $x^2(y^2+1)+y^4=C$. 5.3. $\frac{x^2}{y^3}-\frac{1}{y}=C$. 5.4. $y^4=4xy+C$. 5.5. $x^2+y^2=Cx^3$. 5.6. $\sqrt{x^2-y^2}-x=C$. 5.7. $x+ye^{x/y}=C$. 5.8. $xe^{-y}-y^2=C$.